Hydrodynamical noise and Gubser flow

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Heavy ion collision modeling: state of the art

Heavy-ion collisions are understood in terms of:

- Initial-state fluctuations

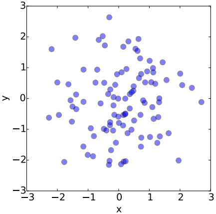


Figure: Random nucleus

- Hydrodynamical evolution

Figure: hydro

Role of fluctuations?

- At experiments, even small systems such as those produced in proton-nucleus collisions seem to produce a fluid.
- Is the ridge seen in proton-proton collisions also due to the formation of a fluid?
  - The question arises:
    What is the role of thermal fluctuations in such small systems?

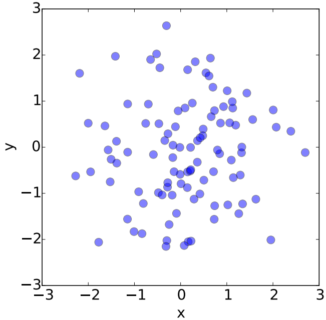
Figure: System whose variations are large when zooming in on a small part.

Two different sources of fluctuations: initial and thermal ones.

Different fluctuations

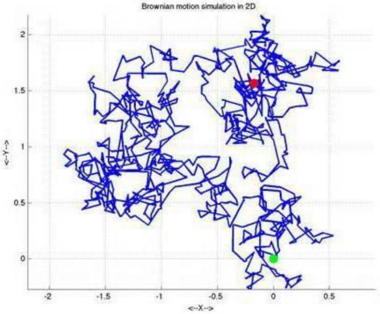
Initial-state fluctuations

- Quantum mechanical origin.



Thermal fluctuations

- Present in all systems with $T > 0$.



Fluctuations in the hydro evolution of a heavy-ion collision

- Hydrodynamics: $\partial_\mu T^{\mu\nu} = 0$ & Equation of state

$$T^{\mu\nu} = T_0^{\mu\nu} + \underbrace{\delta T^{\mu\nu}}_{\text{fluct.}} , \quad \delta T^{\mu\nu} = \begin{cases} \delta T \\ \delta u^\mu \\ S^{\mu\nu} \leftrightarrow \Pi^{\mu\nu} \end{cases}$$

- $T^{\mu\nu}$ is the energy-momentum tensor
- u^μ is the flow four-velocity
- $S^{\mu\nu}$ is the noise tensor
- $\Pi^{\mu\nu}$ is the stress tensor

Solving linearized noisy hydro:

$$\begin{cases} \partial_\mu T_0^{\mu\nu} = 0 & \rightarrow \text{analytically (Bjorken}^1\text{, Gubser)} \\ \partial_\mu \delta T^{\mu\nu} = 0 \end{cases}$$

[1] Kapusta, Müller, Stephanov 2012

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Fluctuation-dissipation theorem

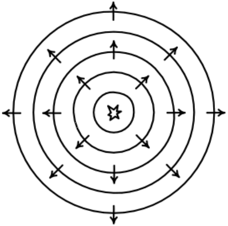
$$\left\langle S^{\mu\nu} S^{\alpha\beta} \right\rangle \sim \frac{\eta}{s} \; \frac{1}{\Delta V \Delta t} \; \Lambda^{\mu\nu\alpha\beta}$$

- Notation:
  - η/s viscosity over entropy ratio
  - $\Delta V \Delta t$ space-time volume
  - $\Lambda^{\mu\nu\alpha\beta}$ unknown tensor
- Thermal fluctuations are related to dissipations.
- Thermal fluctuations are more significant in small systems.
- The structure of $\Lambda^{\mu\nu\alpha\beta}$ may be involved.

Characteristics of Gubser [2] hydro

Exact solution to the equations of hydrodynamics.

- Conformal equation of state $\varepsilon = 3p$
- Boost invariant longitudinal expansion (Bjorken)
- Rotational sym. wrt. the beam axis (p-A and ultra-central A-A)
- Finite transverse size & transverse expansion for $\tau > 0$



[2] Gubser and Yarom 2010

Gubser hydro

Non-trivial analytic solution of hydro with transverse expansion.

- Conformal symmetry
  $\Rightarrow$ hydro eqs are invariant under general coordinate transformations.

  - Uniform fluid at rest in a certain geometry ($dS_3 \times \mathbb{R}$)

$$=$$

Non-uniform expanding fluid in usual laboratory space-time.

- The two space-times are related by coordinate transformations:

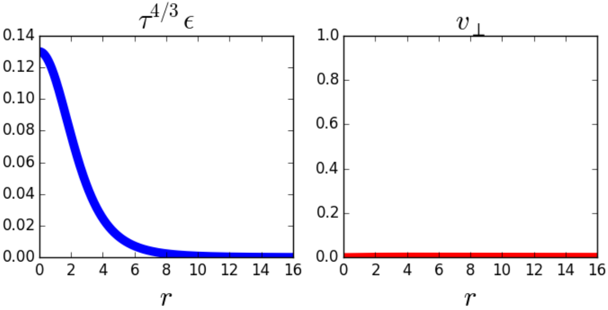
$$(\tau, \xi, r, \phi) \mapsto (\rho, \xi, \theta, \phi)$$

  - τ is the proper time
  - r is the radius in the transverse plane
  - ρ is the de Sitter time
  - ξ is the spatial rapidity
  - $\theta,\ \phi$ are the coordinates on the sphere

  - Denote all quantities in $dS_3 \times \mathbb{R}$ space-time by 'hats'.
  - Rotational $SO(3)$ symmetry in new coordinate system (θ, ϕ).

Evolution of ϵ and $v_\perp$ vs r for Pb-Pb ($\eta/s = 0$)

ϵ =energy density, $v_\perp$ = transverse velocity of fluid



Gubser hydro and thermal noise

- Correlation of thermal noise in Gubser hydro, $(X \to (\rho, \theta, \phi, \xi))$

$$\left\langle \hat{S}^{\mu\nu}(X_1)\hat{S}^{\alpha\beta}(X_2)\right\rangle \propto \frac{\eta}{s}\hat{P}^{\mu\nu}\hat{P}^{\alpha\beta}\delta(X_1 - X_2)$$

  - Tensor structure of $\hat{S}^{\mu\nu}$ is simplified, thanks to $\hat{u}^{\mu} = (1,0,0,0)$.

$$\hat{S}^{\mu\nu}(X) = \hat{w}(\rho)\hat{f}(X)\hat{P}^{\mu\nu}\ , \quad \hat{P}^{\mu\nu} = \mathrm{diag}\,[0,1,1,-2]\ ,$$

  and we have the correlation of the scalar function

$$\left\langle \hat{f}(X_1)\hat{f}(X_2)\right\rangle = \frac{2\nu}{\hat{w}\cosh^2\rho_1 \sin\theta_1}\delta(X_1 - X_2)\ , \quad \nu = \frac{4}{3}\frac{\hat{\eta}}{\hat{s}}$$

  - Magnitude of thermal noise is constrained by $\boldsymbol{\hat{w}} \sim$ **multiplicity**

    Multiplicity more crucial than system size.

    - Same conclusion for Bjorken case.

Noisy Gubser flow

- We solve the Gubser flow mode by mode.
- Decompose thermal fluctuations into scalar and vector modes using spherical symmetry in transformed coordinates $(\rho, \theta, \phi, \xi)$:

$$\delta \hat{T}(\rho,\theta,\phi,\xi) = \hat{T}(\rho) \sum_{l,m} \int \frac{\mathrm{d}k_\xi}{2\pi} \delta_{lm}(\rho, k_\xi) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$

$$\delta \hat{u}_i(\rho,\theta,\phi,\xi) = \sum_{l,m} \int \frac{\mathrm{d}k_\xi}{2\pi} \left[v^s_{lm}(\rho, k_\xi) \partial_i Y_{lm}(\theta,\phi) + v^v_{lm}(\rho, k_\xi) \Phi_{i(lm)}(\theta,\phi) \right] e^{ik_\xi \xi}$$

$$\delta \hat{u}_\xi(\rho,\theta,\phi,\xi) = \sum_{l,m} \int \frac{\mathrm{d}k_\xi}{2\pi} v^\xi_{lm}(\rho, k_\xi) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$

$$\hat{f}(\rho,\theta,\phi,\xi) = \sum_{l,m} \int \frac{\mathrm{d}k_\xi}{2\pi} h_{lm}(\rho, k_\xi) Y_{lm}(\theta,\phi) e^{ik_\xi \xi}$$

Gubser hydro and thermal noise

Each mode evolves according to a Langevin–type equation (from $\partial_\mu \delta T^{\mu\nu} = 0$):

$$\tilde{\mathcal{V}}_l'(\rho) = \underbrace{-\hat{\Gamma}(\rho, l, k_\xi)\tilde{\mathcal{V}}_l(\rho)}_{\text{drag}} + \underbrace{\tilde{\mathcal{K}}(\rho, k_\xi)}_{\text{noise}}$$

$$\tilde{\mathcal{V}}_l(\rho) = \begin{pmatrix} \delta_l(\rho) \\ v_{ls}(\rho) \\ v_{l\xi}(\rho) \\ v_{lv}(\rho) \end{pmatrix}, \quad \tilde{\Gamma} \text{ is a } 4 \times 4 \text{ matrix}, \quad \tilde{\mathcal{K}} = \begin{pmatrix} -\frac{2}{3} \tanh \rho h(\rho) \\ \frac{2\hat{T}}{3\hat{T}'} \tanh \rho h(\rho) \\ -\frac{i4k_\xi \hat{T}}{\hat{T} + H_0 \tanh \rho} h(\rho) \\ 0 \end{pmatrix}$$

- Vector modes are decoupled, and NOT affected by thermal noise.

Solving noisy Gubser flow

- Ultra-central Pb-Pb, p-Pb and p-p.
- The height ($\hat{T}_0$) and width (q^{-1}) of the Gubser solution are determined using the multiplicity and transverse size.

	PbPb	pPb	pp
$\hat{T}_0$	7.3	3.1	2.0
$q^{-1}(fm)$	4.3	1.1	1.1

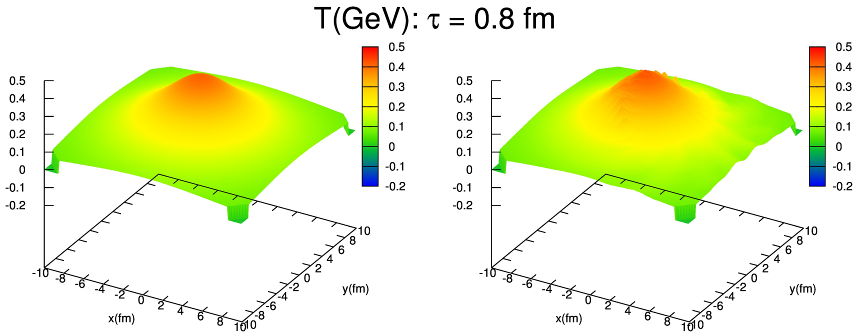
- Approximates system evolution during first several fm's.
- $k_\xi = 0$ mode:
  - Long-range rapidity correlations.
  - Further simplification with v_ξ modes decoupled and indep. of noise $\Rightarrow$ 2 coupled eqs.

Evolution of temperature profile

$T(\tau, \vec{x}_\perp)$ without and with thermal noise, one random event

- x, y are coordinates in the plane transverse to the collision axis.

Pb-Pb



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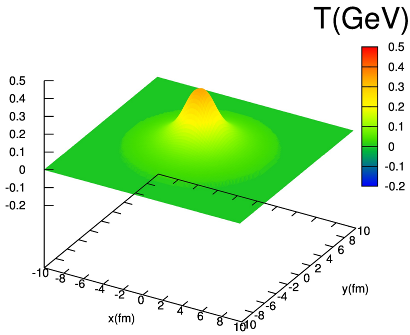
p-Pb



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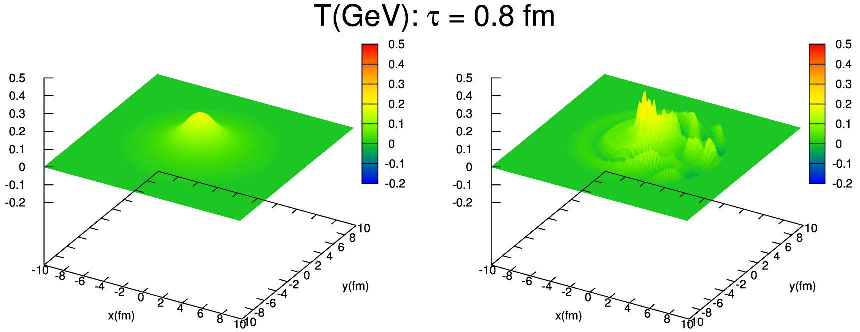
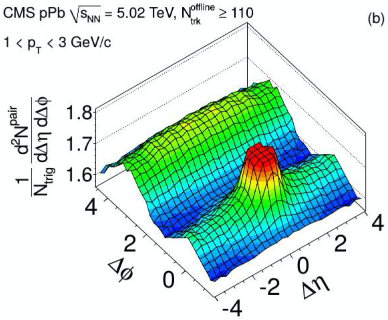
p-p



Hydro & experimental data

- Experimental signature of hydro:
  2-particle correlations averaged over many events.



- Hydro explains the long-range wave structure but not the short-range peak.

- What is the role of hydro fluctuations for the long-range & the short-range correlations?

Making contact with data

Two-particle correlations are not directly calculable in hydro:

- No hadronization or freeze-out.
- Hydro behavior seen through the final particle spectrum.

What we CAN do, however, is to study an object similar to that of two-particle correlations: The two-point correlator of radial flow.

- Radial flow because we are interested in the transverse expansion.

$$\underline{\text{Experiments}} \qquad\qquad \underline{\text{Theory}}$$

$$\langle N(\xi_1, \phi) N(\xi_2, \phi + \Delta\phi) \rangle \quad \sim \quad \langle u_r(r, \phi) u_r(r, \phi + \Delta\phi) \rangle$$

Solving the two-point function of noise

Formal solution:

$$\langle \tilde{\mathcal{V}}_l \tilde{\mathcal{V}}_l \rangle = \text{Initial fluc.} + \text{Thermal fluc.}$$

Initialization of numerics for short-range (1) & long-range (2) initial fluctuations:

$$1) \quad \delta \hat{T}(\theta, \phi, \rho_0, \xi) = \text{const.} \times \delta(\theta - \theta_0)\delta(\phi - \phi_0) = \sum_{l,m} \text{all modes}$$

or

$$2) \quad \delta \hat{T}(\theta, \phi, \rho_0, \xi) = \text{const.} \times \left[(-1)^n \frac{1}{\sqrt{2}} Y_{n,n}(\theta, \phi) + \frac{1}{\sqrt{2}} Y_{n,-n}(\theta, \phi) \right]$$

ε_2(Pb-Pb)~ 0.05, ε_2(p-Pb)~ 0.15 & ε_2(p-p)~ 0.2 $\Rightarrow$ gives const.

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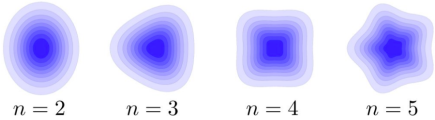

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Radial flow correlation

We choose an observable to characterize the correlation:

$$C_{u_r u_r}(\tau, r, \Delta\varphi) = \langle u_r(\tau, r, \varphi) u_r(\tau, r, \varphi + \Delta\varphi) \rangle - \text{background}$$
$$= C^T_{u_r u_r} + C^I_{u_r u_r}$$

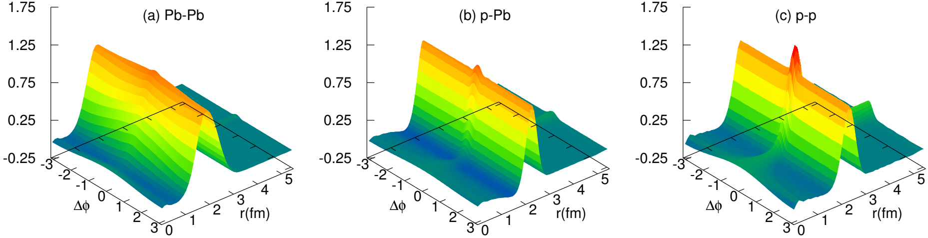
- Equal time (τ) and equal radius (r).
- To see the effect of hydro noise we rescale as follows:

$$\text{Rescaled } C_{u_r u_r} \equiv \frac{C^T_{u_r u_r}}{\text{amplitude of } C^I_{u_r u_r}}$$

- Snapshot at $\tau = 2.5$ fm.

Effect of noise on short-range correlations

$$C_{u_r u_r}(r, \Delta\varphi)$$



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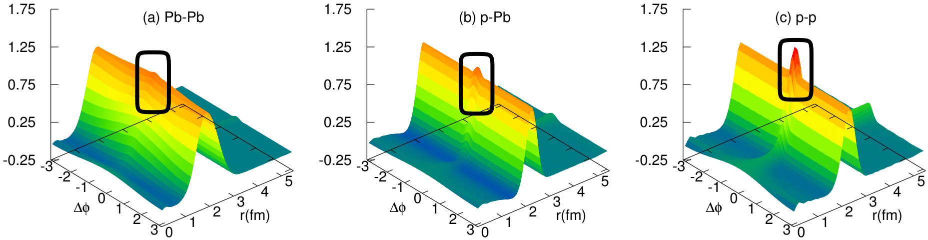
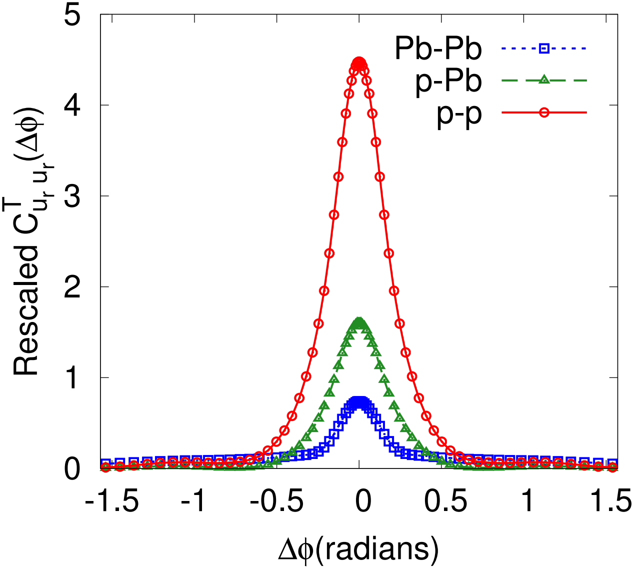
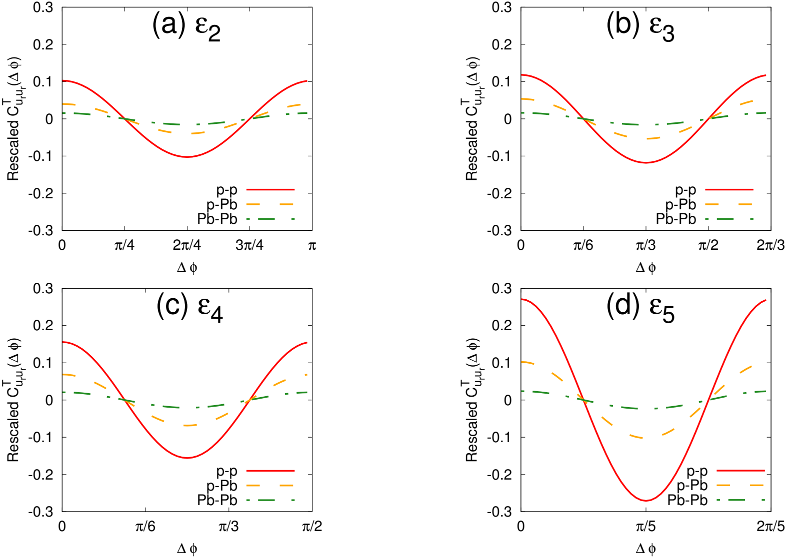


Details of the near-side peak in $C^T_{u_r u_r}$ vs $\Delta\phi$



Effect of noise on long-range correlations



Conclusions

- The absolute amplitude of hydro noise in ultra-central heavy-ion collisions is essentially determined by the multiplicity.
- Long-range correlations $\rightarrow$ evolution of eccentricity:
  - Additional contribution to eccentricity from noise.
  - Stronger in p-p and higher-order harmonics.
  - Effects are NOT sizeable.
- Short-range correlations:
  Noise contributes to the formation of a near-side peak on top of the structure coming from initial state fluctuations:
  - The height and width of the peak grow from Pb-Pb to p-p.

Outlook:

- Longitudinal fluctuations along longitudinal direction, $k_\xi \neq 0$?
- Second order viscous hydrodynamics?

Backup slides

Coordinate transformation

- Weyl rescaling $\mathbb{R}^{1,3} \to \mathrm{dS}_3 \times \mathbb{R}$: $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu}/\tau^2$
  Now the metric reads

$$\mathrm{d}\tilde{s}^2 = \frac{1}{\tau^2}\left(-\mathrm{d}\tau^2 + \mathrm{d}\vec{x}_\perp^2\right) + \mathrm{d}\xi^2 \ .$$

- Reparametrize dS_3 by the mapping $(\tau, r) \to (\rho, \theta)$:

$$\sinh\rho = -\frac{1 - q^2\tau^2 + q^2 r^2}{2q\tau}$$

$$\tan\theta = \frac{2q\tau}{1 + q^2\tau^2 - q^2 r^2}$$

so that the symmetry $SO(1,1) \times \mathcal{Z}_2 \times SO(3)$ is now manifest :

$$\mathrm{d}\hat{s}^2 = \mathrm{d}\rho^2 + \mathrm{d}\xi^2 + \cosh^2\rho\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi\right) \ .$$

Temperatures and multiplicities

$$\hat{T}_0 = \frac{1}{f_*^{1/12}} \left(\frac{3}{16\pi} \frac{\mathrm{d}S}{\mathrm{d}\xi} \right)^{1/3}$$

where $f_* = \epsilon / T^4 = 11$ is extracted from lattice calculations and

$$\frac{\mathrm{d}S}{\mathrm{d}\xi} = 7.5 \frac{\mathrm{d}N_{\mathrm{ch}}}{\mathrm{d}y} \ .$$

- Pb-Pb: $\sqrt{s_{NN}} = 2.76$ TeV corresponding to $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y \sim 1600$
- p-Pb: $\sqrt{s_{NN}} = 5.02$ TeV corresponding to $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y \sim 150$
- p-p: $\sqrt{s} = 13$ TeV corresponding to $\mathrm{d}N_{\mathrm{ch}}/\mathrm{d}y \sim 100$